# 8th Marathon of Parallel Programming WSCAD-SSC/SBAC-PAD-2013

October 24<sup>th</sup>, 2013.

#### **Rules**

For all problems, read carefully the input and output session. For all problems, a sequential implementation is given, and it is against the output of those implementations that the output of your programs will be compared to decide if your implementation is correct. You can modify the program in any way you see fit, except when the problem description states otherwise. You must upload a compressed file with your source code, the *Makefile* and an execution script. The program to execute should have the name of the problem. You can submit as many solutions to a problem as you want. Only the last submission will be considered. The *Makefile* must have the rule *all*, which will be used to compile your source code before submit. The execution script runs your solution the way you design it – it will be inspected not to corrupt the target machine.

All teams have access to the target machine during the marathon. Your execution may have concurrent process from other teams. Only the judges have access to a non-concurrent cluster.

The execution time of your program will be measured running it with *time* program and taking the real CPU time given. Each program will be executed at least twice with the same input and only the smaller time will be taken into account. The sequential program given will be measured the same way. You will earn points in each problem, corresponding to the division of the sequential time by the time of your program (*speedup*). The team with the most points at the end of the marathon will be declared the winner.

This problem set contains 6 problems; pages are numbered from 1 to 19.

# Problem A

## Longest Common Subsequence

A common subsequence of two given sequences may be informally defined as a series of symbols in which all elements are contained in both sequences and appear in the same order. For instance, both "ose" and "os" are common subsequences of "house" and "browse". A Longest Common Subsequence (LCS) of two sequences is a subsequence that has maximum length. Finding a longest common subsequence of two distinct sequences poses a very important challenge to various areas where computer science is applied, namely genetics and speech recognition.

Given a sequence of symbols  $S=\langle a_0, a_1, ..., a_n \rangle$ , a subsequence S' of S is obtained by removing zero or more symbols from S. For example, given  $K=\langle a, b, c, d, e \rangle$ ,  $K'=\langle b, d, e \rangle$  is a subsequence of K. A longest common subsequence of two sequences X and Y is a subsequence of both X and Y with maximum length.

The length of a longest common subsequence, the Levenshtein distance, is a string metric for measuring the difference between two sequences.

The length c[m, n] of a LCS of two sequences  $A=\langle a_0, a_1, ..., a_n \rangle$  and  $B=\langle b_0, b_1, ..., b_n \rangle$  may be defined recursively in the following manner:

$$c[i,j] = \begin{cases} 0 & \text{if} \quad i = 0 \quad \text{or} \quad j = 0, \\ c[i-1,j-1] + 1 & \text{if} \quad i,j > 0 \quad \text{and} \quad a_i = b_j, \\ \max\{c[i-1,j],c[i,j-1]\} & \text{otherwise} \end{cases}$$

From this definition, a dynamic programming algorithm can be derived directly.

Write a parallel program to calculate the size of the Longest Common Subsequence between two sequences. Your solution must use dynamic programming to build a score matrix (as in the provided sequential solution).

#### Input

The program must read two sequences from different files, containing letters and numbers. The first file has the *A* sequence (*fileA.in*). The second file has the *B* sequence (*fileB.in*).

The sequences must be read from the correct input files.

#### **Output**

The program will print the size of the Longest Common Subsequence. You do not need to print the score matrix at the end but the matrix must be built in memory ant it will be used to verify solutions (use the provided debug function to print out the matrix during development).

The output must be written to the standard output.

fileA Input File
heagawghee
fileB Input File
pawheae
Output
5

```
void freeScoreMatrix(mtype **scoreMatrix, int sizeB) {
                                                                                                                                                                                                                                                                                                                     scoreMatrix[i][j-1]);
                                                                                                                                                                                                                                                                                                                                                                                                                    and upper elements*/
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     int LCS(mtype ** scoreMatrix, int sizeA, int sizeB, char * seqA, char *seqB) {
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  void initScoreMatrix(mtype ** scoreMatrix, int sizeA, int sizeB) {
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            mtype **
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               typedef unsigned short mtype;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               allocateScoreMatrix(int sizeA, int sizeB) {
                                                                                                                                                                                     return scoreMatrix[sizeB][sizeA];
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           int i, j;
for (i = 1; i < sizeB + 1; i++) {</pre>
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                for (j = 0; j < (sizeA + 1); j++)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            int i, j;
//Fill first line of LCS score matrix with zeroes
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              for (i = 0; i < (sizeB + 1); i++)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          mtype ** scoreMatrix = (mtype **) malloc((sizeB + 1) * sizeof(mtype *));
                                                          for (i = 0; i < (sizeB + 1); i++)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        for (i = 1; i < (sizeB + 1); i++)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      return scoreMatrix;
free(scoreMatrix);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          //Do the same for the first collumn
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    //Allocate memory for LCS score matrix
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               scoreMatrix[0][j] = 0;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             scoreMatrix[i] = (mtype *) malloc((sizeA + 1) * sizeof(mtype));
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        for (j = 1; j < sizeA + 1; j++) {
   if (seqA[j - 1] == seqB[i - 1]) {</pre>
                           free(scoreMatrix[i]);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          scoreMatrix[i][0] = 0;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 } else {
                                                                                                                                                                                                                                                                                                                                                                                  scoreMatrix[i][j] =
                                                                                                                                                                                                                                                                                                                                                                                                                                                /st else, pick the maximum value (score) from left
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             scoreMatrix[i][j] = scoreMatrix[i - 1][j - 1] + 1;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           /\ast if elements in both sequences match, the corresponding score will be the score from
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    previous elements + 1*/
                                                                                                                                                                                                                                                                                                                                                    max(scoreMatrix[i-1][j],
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       #ifdef DEBUGMATRIX
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    int main(int argc, char ** argv) {
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       locate at the last line and collumn)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                mtype score = LCS(scoreMatrix, sizeA, sizeB, seqA, seqB);
                                                                                                         return EXIT_SUCCESS;
                                                                                                                                                                                  freeScoreMatrix(scoreMatrix, sizeB);
                                                                                                                                                                                                                                                                                               printf("Score: %d\n", score);
                                                                                                                                                                                                                                                                                                                                                                                                                                                printMatrix(seqA, seqB, scoreMatrix, sizeA, sizeB);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           initScoreMatrix(scoreMatrix, sizeA, sizeB);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           mtype ** scoreMatrix = allocateScoreMatrix(sizeA, sizeB);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            // allocate LCS score matrix
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            sizeA = strlen(seqA);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   seqA = read_seq("fileA.in");
seqB = read_seq("fileB.in");
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              char *seqA, *seqB;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       /* if you wish to see the entire score matrix,
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        //fill up the rest of the matrix and return final score (element
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  //initialize LCS score matrix
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       sizeB = strlen(seqB);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    int sizeA, sizeB;
                                                                                                                                                                                                                         //free score matrix
                                                                                                                                                                                                                                                                                                                                      //print score
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             //find out sizes
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           //read both sequences
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        // sizes of both sequences
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                for debug purposes, define DEBUGMATRIX. */
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  / sequence pointers for both sequences
```

# Problem B

### Blum Blum Shub Random Number Generator

Repeatable sequences of random numbers are useful in debugging and to provide reproducibility to experiments. They are also important to guarantee consistency in executions of randomized algorithms, such as Monte Carlo computations. Those sequences are usually provided to programmers in the form of "generators", a stream that provides a random number upon each request.

A random number generator provides new numbers based on its current state. The initial state is given by a seed value. A generator f generates ith random number at the ith request in function of the (i-1)th state, which is updated to the ith state, used in the same fashion by later requests.

The Blum Blum Shub random number generator was proposed in 1986 by Leonel Blum, Manuel Blum and Michael Shub. It is advised to be used in cryptography, but not in simulations, because the required modulus operations are slow. In what follows, its seed is denoted by  $x_0$ . Number  $x_i$ , the *i*th number on the sequence, is obtained by performing:

$$x_i = (x_{i-1})^2 \bmod M$$

where M = pq is the product of two primes. It is possible to generate the *i*th number (thus setting generator to the *i*th state) directly from the seed by Euler's Theorem:

$$x_i = \left(x_0^{2^i \bmod \lambda(M)}\right) \bmod M$$

where  $\lambda(M) = \text{lcm}((p-1), (q-1))$ , function lcm returning the least common multiplier between two integers.

You should write a program that generates in parallel the N first random numbers of a seeded Blum Blum Shub generator.

#### Input

It is a line in the form:

Where  $\langle seed \rangle$  is the initial state value  $x_0$ , numbers  $\langle p \rangle$  and  $\langle q \rangle$  are the primes used to calculate M and  $\langle N \rangle$  is amount of random numbers that shall be generated by the Blum

Blum Shub generator from  $x_0$ .

The input must be read from the standard input.

## Output

It contains just one line, with all generated numbers separated by space.

The output must be written to the standard output.

Input	Output for the input	
3 11 19 9	9 81 82 36 42 92 104 157 196	

```
T lcm (T x, T y)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        // Calculates Least Commom Multiple. 
// uses eq: lcm (x, y) * gcd (x, y) == xy template <typename T>
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         // T models integer
T gcd (T x, T y)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        class BlumBlumShub {
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            // Blum, Lenore; Blum, Manuel; Shub, Mike (1 May 1986).
// "A Simple Unpredictable Pseudo-Random Number Generator".
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      // Reference:
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 // Implementation of Blum Blum Shub generator for integers.
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   template <typename T>
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   // SIAM Journal on Computing 15 (2): 364–383.
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              // Calculates Greatest <a href="Commom Divisor">Commom Divisor</a>
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           T tmp (gcd (x, y));
return tmp ? (x * y / tmp) : T (0);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    while (true) {
   if (x == T (0)) return y;
                                                      BlumBlumShub (size_t seed, size_t p, size_t q)
                                                                                                                                                   public:
                                                                                                                                                                                                                                                                                                                                                                            void set_state (size_t seed, size_t p, size_t q)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               size_t seed, p, q, x;
                                                                                            // constructor
                                                                                                                                                                                                                                                                                                                                                                                                                                     // Sets state.
                                                                                                                                                                                                                                   this->q = q;
this->x = seed;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           if (y == T(0)) return x;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           y %= x ;
  set_state (seed, p, q);
                                                                                                                                                                                                                                                                                                                       this->seed = seed ;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  х %= у ;
                                                                                                                                                                                                                                                                                         this->p=p;
                                                           void generate (R& f, size_t* i, size_t* j)
                                                                                                                     template <typename R>
                                                                                                                                           // Fills range [i,j) with random numbers generated by
                                                                                         // R models generator function
for ( ; i != j ; ++ i) *i = f () ;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             BlumBlumShub (const BlumBlumShub& g)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 size_t operator() (void)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               // assignment operator
BlumBlumShub& operator= (const BlumBlumShub& g)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                 size_t operator() (const size_t i)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          BlumBlumShub ()
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     // this small seed and prime factors generate large cycles
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   // default constructor
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    // (uses Euler's Theorem)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          // May be useful to jump to a desired distance...
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  // Generates next random number and updates state
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           // copy-constructor
                                                                                                                                                                                                                                                                                                                       \times %= p * q;
                                                                                                                                                                                                                                                                                                                                                                              size_t e (pow (2, i));
e %= 1cm (p - 1, q - 1);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              set_state (size_t (3), size_t (11), size_t (19));
                                                                                                                                                                                                                                                                                                                                                   x = size_t (pow (seed, e));
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    return x ;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          x = (x * x) % (p * q);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    return *this;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          if (this != &g) new (this) BlumBlumShub (g);
                                                                                                                                                                                                                                                                                              return x ;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         set_state (g.seed, g.p, g.q);
```

# Problem C

# The Knight's Tour Problem

The *Knight's Tour Problem* is a classical mathematical problem which dates back to the 9<sup>th</sup> century.

In the chess game, a knight (((i))) may only make moves which simultaneously shift one square along one axis and two along the other (as illustrated in Figure C1a). A knight's tour on a chessboard (or any other grid of any size) is a sequence of moves by a knight such that each square of the board is visited exactly once.

Write a parallel program that finds a knight's tour starting at a given position, in a square grid of size  $s \times s$ .

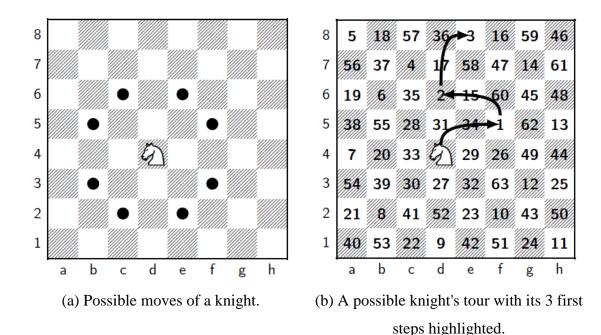


Figure C1. One of the possible knight's tours starting at position d4.

#### Input

The input contains only one test case. The first (and only) line contains three integers: the size  $s \ge 5$  of the side of the square, and a pair of coordinates x and y with the initial position of the knight. To simplify, assume that each axis is numbered from  $\theta$  up to s-1. The input must be read from the standard input.

## Output

The output must be an  $s \times s$  table, where each cell contains the number of knight movements to reach it.

The output must be written to the standard output.

Input	Output for the input
5 0 0	0 13 18 7 24 5 8 1 12 17 14 19 6 23 2 9 4 21 16 11 20 15 10 3 22

```
const int move[8][2]={1,2, 2,1, 2,-1, 1,-2, -1,-2, -2,-1, -2,1, -1,2};
                                                                                                                                                                                                                                                                                                                                                                   // Copy constructor
tour(const tour&T): sx(T.sx), sy(T.sy), size(T.size) {
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        bool findtour(tour& , int );
                                                       // Function to output class to ostream
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   tour(int s = 5, int startx = 0, int starty = 0)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                int sx, sy, size;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              vector< vector< int > > board;
(std::ostream& os, const tour& T);
                              friend std::ostream& operator<<</pre>
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      // Constructor
                                                                                                                                       // Copy the board
for(int i = 0; i < size; ++i)
for(int j = 0; j < size; ++j)
board[i][j] = T.board[i][j];;</pre>
                                                                                                                                                                                                                                                                                                                      for(int i = 0; i < size; ++i)
                                                                                                                                                                                                                                                                                                                                                  this->board.resize(size);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       if(!findtour(*this, 0))
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          board[sx][sy] = 0;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       board.resize(size);
for(int i = 0; i < size; ++i)</pre>
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          :sx(startx), sy(starty), size(s)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        for(int i = 0; i < size; ++i)</pre>
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       // Fill the board with -1s
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          // Solve the problem
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          // Move 0
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   // Get the board to size x size
                                                                                                                                                                                                                                                                                        board[i].resize(size);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                cout << "No solutions found\n";</pre>
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          for(int j = 0; j < size; ++j)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                board[i].resize(size);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               board[i][j] = -1;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                bool tour::findtour(tour& T, int imove) {
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 std::ostream& operator<<(std::ostream& os, const tour& T) {</pre>
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              // A recursive function to find the knight tour.
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               for(int i = 0; i < size; ++i) {
   for(int j = 0; j < size; ++j)
   os << setw(2) << T.board[i][j] << " ";</pre>
                                                                                                                return false;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 for (int i = 0; i < 8; ++i) {
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           int cs = T.size;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    int cy = T.sy;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     int cx = T.sx;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   // make a move
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       if(imove == (size*size - 1)) return true;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      return os;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       int size = T.size;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        int tcx = cx + move[i][0];
int tcy = cy + move[i][1];
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         os << endl;
                                                                                                                                                                                                                                                                                                                    if(findtour(temp, imove+1)) {
                                                                                                                                                                                                                                                                                                                                                                                                    temp.board[tcx][tcy] = imove+1;
                                                                                                                                                                                                                                                                                                                                                                                                                                  tour temp(T);
                                                                                                                                                                                                                                                                                                                                                  temp.sy = tcy;
                                                                                                                                                                                                                                                                                                                                                                           temp.sx = tcx;
                                                                                                                                                                                                                                                               exit(0);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   (tcx >= 0) \&\& (tcy >= 0) \&\& (tcx < cs) \&\& (tcy < cs) \&\&
                                                                                                                                                                                                                                                                                           cout << temp << endl;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           (T.board[tcx][tcy] == -1)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            // Has this place been visited yet?
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   // Is this a valid move?
```

# Problem D

## Selection

The problem consists in selecting the  $k^{th}$ -largest element from an unsorted list. One obvious solution is to first sort the elements of the list and then pick out the  $k^{th}$  position. If you need to put out different  $k^{th}$  items multiple times, it might be worthwhile to do the sorting. If, however, you only need to this once, or if the list is updated between selections, you can use an algorithm with a better asymptotic complexity than sorting, namely O(n).

The proposed Selection algorithm to accomplish this task recursively divides the original list in such a way that the search only occurs in subsequences. The central point of this algorithm is in finding the median of a list (the item in the middle). The recursive serial algorithm for selection can be described with five steps:

- 1. If the size of the data set to be used is less than some constant small size, *Q*, sort the data and return the k<sup>th</sup> element; otherwise, subdivide the data set into chunks of size *Q* and whatever is left over.
- 2. Sort each chunk and find the median of the medians found in the previous step, creating a new array of medians.
- 3. Recursively call the selection routine to find the median of the medians in the array of medians found in the previous step.
- 4. Partition the original data set into three subsequences: those whose elements are less than the median of medians, those that are equal to the median of medians and those that are greater than the median of medians.
- 5. Determine which subsequence contains the  $k^{th}$  element, from the sizes of the three subsequences, and recursively call the selection routine on that subsequence. If the  $k^{th}$  element is not in the subsequence of smaller or larger items, it must be in the subsequence equal to the median of medians, so just return the medians of medians value.

According to some authors, the value of Q can be any integer greater than or equal to 5. In the example below this value is set to 5. In the example, S is an array of integers of

size *N*.

There are four support functions in the algorithm:

- SortSelect5(): sorts each chunk of Q = 5 elements and returns the median of the sorted data (not shown).
- **SortLessThanQ():** if there is a leftover chunk of less than Q elements (*lastNum*), the median of this chunk is found by calling this function (**not shown**).
- CountAndMark(): takes an array of data to be partitioned (S), the array that will hold the notation of which partition the corresponding element from S will be assigned (Marks), the number of elements in the first two parameters arrays (num), the value of the median that determines the three partitions (median), and the leg array to hold the counts of the number of elements that are less than, equal to, or greater than median.
- ArrayPack(): takes a list of elements (S) and an array of items, noting which partition the element from S should be assigned (Marks). Everywhere the Marks element and the scanSym match, the corresponding element of S is packed into sPack (tip: is possible to do this operation with a prefix scan of the Marks array).

#### Input

The input contains only one test case. The first line contains two integers that correspond respectively to k (the position to be selected) and N (the size of the list). The next lines contain the numbers of the unsorted list, one number in each line. Your program must accept lists up to 500,000,000 of integer elements.

The input must be read from the standard input.

#### Output

The output contains the k<sup>th</sup>-largest element from the unsorted list.

The output must be written to the standard output.

12

Input	Output for the input
6 10	5
2	
1	
8	
3	
9	
6	
4	
3	
7 0 5	

```
int SequentialSelect(int *S, int num, int k)
{
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    int SortLessThanQ(int S[], int num,
{
                                                                                                                                                                                                                                                                                                                         }

If the column-;

int M = SequentialSelect(Medians, cNum, (cNum+1)/2);

delete Medians;

int leg[3] = (0,0,0);

int *markS = new int[num];

int *markS = new int[num];
                                                                                                                                                                                                                                                     } else {
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  int lastNum = num - (Q * (num / Q));
if (lastNum) {
    if (lastNum) {
        if lastQ = Q * (num / Q);
        Medians[cNum-1] = SortLessThanQ(&S[lastQ], lastNum, (lastNum+1)/2);
        if (lastNum-1) = SortLessThanQ(&S[lastQ], lastNum, (lastNum-1)/2);
        if (lastNum-1) = SortLessThanQ(&S[lastQ], lastNum-1)/2;
        if (lastNum-1) = SortLessThanQ(&S[lastQ], lastNum-1)/2;
        if (lastNum-1) = SortLessThanQ(&S[lastQ], lastNum-1)/2;
        if (lastNu
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 int cNum = num/Q + 1;
int *Medians = new int[cNum];
int i = 0;
for (int j = 0; j < num/Q; j+</pre>
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            CountAndMark(S, markS, num, M, leg);
if (leg[0] >= k) {
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  if (num <= Q) return SortLessThanQ(S, num, k);</pre>
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         switch (num)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          (int j = 0; j < num/Q; j++) { Medians[j] = SortSelectS(&S[i], 3); // find medians of subsequences
M = SequentialSelect(sPack, leg[2], k-(leg[0]+leg[1]));
delete sPack;
return M;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                M = SequentialSelect(sPack, leg[0], k);
delete sPack;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 i += Q;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    case 2:
                                                                                                                                                                     ArrayPack(S, sPack, num, markS, 2);
                                                                                                                                                                                                               int *sPack = new int[leg[2]];
                                                                                                                                                                                                                                                                                               return M;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     delete markS;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       ArrayPack(S, sPack, num, markS, 0);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   case 5:
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   case 4:
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        case 3:
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     case 1:
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     int *sPack = new int[leg[0]];
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    printf("SortLessThanQ Error: num not in [1..4]
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        return SortSelect5(S, k); break;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             return SortSelect4(S, k); break;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         return SortSelect3(S, k); break;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          return SortSelect2(S, k); break;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              return S[0]; break;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              int
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    <u>num</u> = %d\n",num);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       }
void CountAndMark(int S[], int Marks[], int num, int median, int leg[3])
{
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              int SortSelect5 (int S[], int k)
{
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         }
int SortSelect4 (int S[], int k)
{
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     }
int SortSelect3 (int S[], int k)
{
                                                                                                                                                                                                                                                     void ArrayPack(int S[], int sPack[], int
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     int SortSelect2 (int S[], int k)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         #define swap(A,B) {int t; t = A; A = B; B = t;}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    if (S[0] > S[1]) swap(S[0], S[1]);
if (S[1] > S[2]) swap(S[1], S[2]);
if (S[0] > S[1]) swap(S[0], S[1]);
if (S[2] > S[3]) swap(S[0], S[3]);
if (S[1] > S[3]) swap(S[1], S[3]);
if (S[0] > S[1]) swap(S[0], S[1]);
if (S[0] > S[1]) swap(S[0], S[1]);
if (S[0] > S[1]) swap(S[0], S[1]);
if (S[1] > S[3]) swap(S[3], S[4]);
if (S[1] > S[3]) swap(S[1], S[3]);
if (S[1] > S[3]) swap(S[1], S[3]);
if (S[1] > S[3]) swap(S[1], S[3]);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   if (S[0] > S[1]) swap(S[0], S[1]);
if (S[1] > S[2]) swap(S[1], S[2]);
if (S[0] > S[1]) swap(S[0], S[1]);
if (S[2] > S[3]) swap(S[2], S[3]);
if (S[1] > S[2]) swap(S[1], S[2]);
if (S[0] > S[2]) swap(S[0], S[1]);
    int i, j=0;
for (i = 0; i < num; i++)
    if (Marks[i] == scanSym) sPack[j++] = S[i];</pre>
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               if (S[0] > S[1]) swap(S[0], S[1]);
if (S[1] > S[2]) swap(S[1], S[2]);
if (S[0] > S[1]) swap(S[0], S[1]);
return S[k-1];
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               if (S[0] > S[1]) swap(S[0],
if (k == 1) return S[0];
else return S[1];
                                                                                                                                                                                                                                                                                                                                                                                                              for (int i = 0; i < num; i++) {
    if (S[i] == median) {Marks[i] = 1; leg[1]++;}
    else if (S[i] < median) {Marks[i] = 0; leg[0]++;}
    else {Marks[i] = 2; leg[2]++;}</pre>
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 return S[k-1];
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      return S[k-1];
                                                                                                                                                                                                                                                     'wnu
                                                                                                                                                                                                                                                     int Marks[], int scanSym)
```

# Problem E

# Black-Scholes<sup>1</sup>

A *European call option* gives its holder the opportunity to purchase from the writer an asset at an agreed expiry time T at an agreed exercise price E. Given a time t, we will let S(t) denote the asset value at time t, so S(T) is the value of the asset at the expiry time. The final payoff to the purchaser is  $\max\{S(T) - E, 0\}$ , because

- if S(T) > E, the option will be exercised for a profit of S(T) E, whereas
- if  $S(T) \le E$ , the option will not be exercised.

In 1973, Robert C. Merton published a paper presenting a mathematical model which can be used to calculate a rational price for trading options. In that same year, options were first traded in the open market. Since then, the demand for option contracts has grown to the point that trading options typically far outstrips that for the underlying assets. Merton's work expanded on that of two other researchers, Fischer Black and Myron Scholes, and the pricing model became known as the *Black-Scholes model*. The model depends on a constant  $\sigma$  representing how volatile the market is for the given asset, as well as the continuously compounded interest rate r.

Write a parallel version<sup>2</sup> of this problem that uses the Monte Carlo technique to calculate the Black-Scholes pricing model to a set of assets.

#### Input

The input contains only one test case. The first line contains only one integer: the number of assets N ( $1 \le N \le 32.768$ ).

The input must be read from the standard input.

#### **Output**

The output contains the asset price, each one in separate line.

The output must be written to the standard output.

<sup>&</sup>lt;sup>1</sup> Homework 1: A Parallel Monte Carlo Simulation for Black-Scholes Option Valuation. URL: <a href="http://www.cs.berkeley.edu/~yelick/cs194f07/hw/hw1/">http://www.cs.berkeley.edu/~yelick/cs194f07/hw/hw1/</a> at May, 2013.

<sup>&</sup>lt;sup>2</sup> The serial version came from Intel® Development Forum – Brazil 2012.

Input	Output for the input
10	17.07
	7.19
	25.40
	1.23
	5.97
	11.77
	22.74
	0.01
	1.88
	8.17

```
const int RAND_N = 1 << 18;
static const float RISKFREE = 0.06;
static const float VOLATILITY = 0.10;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       #ifndef max
#define max(a,b) (((a) > (b)) ? (a) : (b))
                                                                                                                                                                                                                                                                                                 c9)))))));
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         Basetype cdfnorminv(Basetype P) {
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     union FPARRAY {
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            template<class Basetype>
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           z = y * (((a4 * z + a3) * z + a2) * z + a1) / ((((b4 * z + b3) * z + b2) * z + b1) * z + 1);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  const Basetype b1 = -8.4735109309;
const Basetype b2 = 23.08336743743;
const Basetype b3 = -21.06224101826;
const Basetype b4 = 3.13082909833;
const Basetype c1 = 0.337475482272615;
const Basetype c2 = 0.976169019091719;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   y = P - 0.5;
if (fabs(y) < 0.42) {
z = y * y;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  if (P <= 0 || P >= 1.0) { printf("MoroInvCND(): bad parameter\n");
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         const
                                             return z;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     Basetype y, z;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   const Basetype c9 = 3.960315187E-07;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                const Basetype c8 = 2.888167364E-07;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                const
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       const Basetype c4 = 2.76438810333863E-02;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    const Basetype c3 = 0.160797971491821;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   const Basetype a4 = -25.44106049637;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            const Basetype a3
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      const Basetype a2 = -18.61500062529;
const Basetype a3 = 41.39119773534;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 const Basetype a1 = 2.50662823884;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          double *DPData;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             float *SPData;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   Basetype c5 = 3.8405729373609E-03;
Basetype c6 = 3.951896511919E-04;
Basetype c7 = 3.21767881768E-05;
                                                                                                                                                                                                                                                   뱎
                                                                                                                                                                                                                                                                                                                                                    z = c1 + z * (c2 + z * (c3 + z * (c4 + z * (c5 + z * (c6 + z * (c7 + z * (c8 + z * (c4 + z * (c5 + z * (c6 + z * (c7 + z * (c8 + z * (
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     if (y > 0)
                                                                                                                                                                                                                                                   (y < 0)
                                                                                                                                                                                                                                                                                                                                                                                                       z = log(-log(P));
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       z = log(-log(1.0 - P));
                                                                                                                                                                                                      -z;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    int main(int argc, char* argv[]) { FPARRAY CallResultParallel, CallConfidence, StockPrice, OptionStrike,
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              (float) (RAND_N - 1)));
h_CallConfidence[opt] = (float) (exprt * 1.96 * stdDev / sqrtf((float) RAND_N));
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     void MonteCarlo(float *h_CallResult, float *h_CallConfidence, float
float *X, float *T, int OPT_N) {
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 1_Random[pos]) - Xval);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               int i, mem_size;
int OPT_N;
scanf("%d", &OPT_N);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      for (int opt = 0; opt < OPT_N; opt++) {
   float VBySqrtT = VOLATILITY * sqrt(T[opt]);
   float MuByT = (RISKFREE - 0.5 * VOLATILITY * VOLATILITY) * T[opt];</pre>
                                                                                                                                                   // results in CallResultParallel.SPData[i]
for (i = 0; i < OPT_N; i++)</pre>
                                                                                                                                                                                                                                                                                                 MonteCarlo(CallResultParallel.SPData, CallConfidence.SPData, StockPrice.SPData, OptionStrike.SPData,
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        for (i = 0; i < OPT_N; i++) {
    CallResultParallel.SPData[i] = 0.0;</pre>
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         } //end of for
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 mem_size = sizeof(float) * OPT_N;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               for (int k = 0; k < RAND_N; k++)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            float 1_Random[RAND_N];
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 ...; // malloc commands
...; // free commands
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  CallConfidence.SPData[i] = -1.0;
StockPrice.SPData[i] = RandFloat(5.0f, 50.0f);
OptionStrike.SPData[i] = RandFloat(10.0f, 25.0f);
OptionYears.SPData[i] = RandFloat(1.0f, 5.0f);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   float exprt = expf(-RISKFREE * T[opt]);
h_CallResult[opt] = exprt * val / (float) RAND_N;
float stdDev = sqrtf(((float) RAND_N * val2 - val * val) / ((float) RAND_N *
                                                                                                printf("%5.2f\n", CallResultParallel.SPData[i]);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           float val = 0.0, val2 = 0.0;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     float Xval = X[opt];
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             float Sval = S[opt];
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       l_Random[k] = cdfnorminv<float>((k + 1.0) / (RAND_N + 1.0));
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           OptionYears;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            val += callValue;
val2 += callValue * callValue;
                                                                                                                                                                                                                                                                                                      OptionYears.SPData, OPT_N);
```

# Problem F

#### Gauss

The solution of sets of linear equations has applications on electrical circuit analysis, simulations of network traffic, simulations of neurons and in various other fields such as chemistry, physics, biology, economics, engineering and social sciences. In this exercise you must solve a system of linear equations given by Ax=b, where  $A=\{A_{11}, A_{12}, ..., A_{nn}\}$  is a matrix of size  $n \times n$ , and  $x=\{x_1, x_2, ..., x_n\}$  and  $b=\{b_1, b_2, ..., b_n\}$  vectors of length n. To solve a linear system, the program first runs the Gaussian elimination algorithm, which transforms the matrix A into an upper triangular matrix. To this end, for each line i, the algorithm transform in zero all elements from column i for the lines i+1 to n. This is accomplished by summing each element from row i to each elements of each row j below it, multiplied by the factor  $-A_{ii}/A_{ji}$ .

Once transformed into an upper triangular matrix, one can solve the linear system by evaluating the value  $x_n = b_n/A_{nn}$ . Then we can calculate  $x_{n-1}$  and so on.

Your program should generate the linear system using the function  $generateLinearSystem(int\ n,\ float\ *A,\ float\ *b,\ int\ nS)$ , which generates nS matrices of size  $n \times n$  and a vector b, filled with n values 1. In addition, each solution found is tested using the function  $testLinearSystem(float\ *A,\ float\ *b,\ float\ *x,\ int\ n,\ int\ nS)$ , which replaces the solutions x of each of the nS linear systems, checking if Ax=b.

#### **Input**

The input contains only one test case. The first line contains two integers: the size of each linear system N and the number of linear systems to be solved nS, respectively  $(N \ge 1.000; nS \ge 30)$ 

The input must be read from the standard input.

## Output

The output shows the amount of errors found during the test phase for each line linear system.

The output must be written to the standard output.

Input	Output for the input
1000 30	Errors=0

```
1);
                                                                                                                                                                                                                                                                                                                                                                                                       void solveLinearSystem(const float *A, const float *b, float *x,
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             void generateLinearSystem(int n, float *A, float *b, int nS) {
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      int testLinearSystem(float *A, float *b,
                                                                                                                                                                                                                                                                float *bcpy = (float *) malloc(n * sizeof(float));
memcpy(Acpy, A, n * n * sizeof(float));
                                                                                                                                                                                                                                          memcpy(bcpy, b, n * sizeof(float));
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               int i, j;
for (i = 0; i < n; i++) {</pre>
                                                                                                                                                                      int i, j, count;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 for (i = 0; i < n; i++)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              for (i = 0; i < n; i++) {
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             return 0;
                                                                                                         for (i = 0; i < (n - 1); i++) {
                                                                                                                                                                                                                                                                                                                                     float *Acpy = (float *) malloc(n * n * sizeof(float));
                                                                                                                                             /* Gaussian Elimination */
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               for (j = 0; j < n; j++)

A[i * n + j] = (1.0 * n + (rand() % n)) / (i + j + j)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    for (j = 0; j < n; j++)

sum += A[i * n + j] * x[j];
                                       for (j = (i + 1); j < n; j++) {
    float ratio = Acpy[j * n + i] / Acpy[i * n + i];</pre>
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       if (abs(sum - b[i]) >= 0.001) {
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 A[i * n + i] = (10.0 * n) / (i + i + 1);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           float sum = 0;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            return 1;
              for (count = i; count < n; count++) {</pre>
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      float *x, int n, int nS) {
                                                                                                                                                                                                                                                                                                                                                                                                          int n) {
                                                                                               n], n, nS);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   int main(int argc, char **argv) {
                                                                                                                                                                                                                                                                                                                                                                                                              float *A = (float *) malloc(nS * n * n * sizeof(float));
float *b = (float *) malloc(nS * n * sizeof(float));
float *x = (float *) malloc(nS * n * sizeof(float));
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     x[n - 1] = bcpy[n - 1] / Acpy[(n - 1) * n + n - 1];
for (i = (n - 2); i >= 0; i--) {
    float temp = bcpy[i];
    for (j = (i + 1); j < n; j++) {</pre>
                               printf("Errors=%d\n", nerros);
                                                                                                                                                                 for (i = 0; i < nS; i++)
                                                                                                                                                                                                                                                                                                                                                    for (i = 0; i < nS; i++)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         int i, nerros = 0;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    scanf("%d %d", &n, &nS);
return EXIT_SUCCESS;
                                                                                                                                                                                                                                                         for (i = 0; i < nS; i++)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        int n, nS;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               /* Back-substitution */
                                                                                                                                                                                                                         solveLinearSystem(&A[i * n * n], &b[i * n], &x[i * n], n);
                                                                                                                                                                                                                                                                                                                 generateLinearSystem(n, &A[i * n * n], &b[i * n], nS);
                                                                                                                                 nerros += testLinearSystem(&A[i * n * n], &b[i * n], &x[i *
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           x[i] = temp / Acpy[i * n + i];
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           temp -= (Acpy[i * n + j] * x[j]);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         bcpy[j] -= (ratio * bcpy[i]);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      Acpy[j * n + count] -= (ratio * Acpy[i * n +
```