14th Marathon of Parallel Programming SBAC-PAD & WSCAD – 2019

October 17th, 2019.

Rules for Local Contest

For all problems, read carefully the input and output session. For all problems, a sequential implementation is given, and it is against the output of those implementations that the output of your programs will be compared to decide if your implementation is correct. You can modify the program in any way you see fit, except when the problem description states otherwise. You must upload a compressed file (zip) with your source code, the *Makefile* and an execution script. The script must have the name of the problem. You can submit as many solutions to a problem as you want. Only the last submission will be considered. The *Makefile* must have the rule *all*, which will be used to compile your source code. The execution script runs your solution the way you design it – it will be inspected not to corrupt the target machine.

All *Local Teams* must use the computers that the organization provides. Only the judges have access to the judge machine.

The execution time of your program will be measured running it with *time* program and taking the real CPU time given. Each program will be executed at least three times with the same input and the mean time will be taken into account. The sequential program given will be measured the same way. You will earn points in each problem, corresponding to the division of the sequential time by the time of your program (*speedup*). The team with the most points at the end of the marathon will be declared the winner.

General information

Compilation

You must use CC or CXX inside your *Makefile*. Be careful when redefining them! There is a simple *Makefile* inside you problem package that should be modified. Example:

```
FLAGS=-O3
EXEC=sum
CXX=icpc
all: $(EXEC)

$(EXEC):
    $(CXX) $(FLAGS) $(EXEC).cpp -c -o $(EXEC).o
    $(CXX) $(FLAGS) $(EXEC).o -o $(EXEC)
```

Running

You must have an execution script that has the same name of the problem. This script runs your solution the way you design it. There is a simple script inside you problem package that should be modified. Example:

```
$ cat A
#!/bin/bash
# This script runs Problem A
# export OMP_NUM_THREADS=32
# mpiexec -n 32 ./sum
./sum
```

Measure the execution time of your solution using *time* program. Add input/output redirection when collecting time. Use *diff* program to compare the original and your solution results. Example:

```
$ time -p ./A < original_input.txt > my_output.txt
real 4.94
user 0.08
sys 1.56
$ diff my_output.txt original_output.txt
$
```

Problem A

Maximum Sum Subsequence

Given an array sequence $[A_1, A_2 ... A_n]$, where n is the total amount of integer values (also the array size), this implementation aims to find the maximum possible sum of increasing subsequence S of length k such that $S_1 \le S_2 \le S_3 \le S_4 \le \le S_k$.

Input

The input set contains only one test case. The first line contains one value: the array size (which is also the amount of elements to be read). The second line contains the length of the subsequence S, represented by k. Then, the last line contains a list of the elements to be inserted into the array (note that the number of elements must be equal to the array size). The input must be read from the standard input.

Output

The output contains only one line printing the maximum possible sum of increasing subsequence S.

The output must be written to the standard output.

Input example 1	Output example 1
8 3 8 5 9 10 5 6 21 8	40

```
int MaxIncreasingSub(int arr[], int n, int k)
{
    int **dp, ans = -1;
    dp = new int *[n];
    for(int i = 0; i < n; i++)
    dp[i] = new int[k+1];
    for(int i = 0; i < n; i++) {
        for(int i = 0; i < n; i++) {
            dp[i][1] = -1;
        }
    for (int i = 1; i < n; i++) {
            dp[i][1] = arr[i];
        }
    for (int j = 0; j < i; j++) {
            for (int j = 0; j < i; j++) {
                if (arr[j] < arr[i]) }
        }
        roundle for (int i = 1, i < n; i++) {
                if (dp[j][1] != -1) {
                  dp[i][1] + arr[i]);
        }
        }
    }
    return (ans == -1) ? 0 : ans;
}
</pre>
```

Problem B

Brute-Force Password Cracking

One way to crack a password is through a brute-force algorithm, which tests all possible combinations of a password exhaustively until it finds the correct one.

A common practice in security systems is to store the *hash* of user passwords (instead of the plan text). Hashes map a given x (the password) with variable length N to a given y = hash(x) with a fixed length M, following specific security properties. One of them is: you cannot obtain the value of x from the value of y. In a security system, y is stored instead of x and the comparison is performed directly between the hash of attempted password x' and the stored y = hash(x).

Examples of hash algorithms are MD5 and SHA. For MD5 algorithm, the fixed length of the hash(x) is 128 bits usually presented as 32-hexadecimal characters.

This problem considers the cracking of MD5-hashed password by trying exhaustively all the combinations of possible characters (uppercase and lower-case letters and numeric symbols) and comparing pairs of MD5 hashes. That is, for all combinations of characters x'_i , $hashMD5(x) = hashMD5(x'_i)$?

Input

An input consists of only one case of test. The single line contains a string with 32-hexadecimal characters representing the value of MD5 hash of the password to be cracked. Consider that the possible passwords used to generate the hashes have the length N, with $1 \le N \le 10$.

The input must be read from the standard input.

Output

The output is a single line that contains the password value found.

The output must be written to the standard output.

Input example 1	Output example 1
7a95bf926a0333f57705aeac07a362a2	found: PASS

```
윳);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            int len, int *ok) {
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            void print_digest(byte * hash){
                             MD5_DIGEST_LENGTH) == 0){
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        void iterate(byte * hash1, byte * hash2, char *str, int idx,
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       letters
                                                                                                                                                                                                                   hashes.
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 * This procedure generate all combinations of possible
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     if(*ok) return;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              int c;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       printf("\n");
                                                                                                                                                                                                                                                                            } else ·
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      if (idx < (len - 1)) {</pre>
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   for(x = 0; x < MD5_DIGEST_LENGTH; x++)</pre>
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  int x;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               // 'ok' determines when the algorithm matches.
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       printf("%02x", hash[x]);
                                                                                                                                                                                  for (c = 0; c < strlen(letters) && *ok==0; ++c)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          for (c = 0; c < strlen(letters) && *ok==0; ++c)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            // Iterate for all letter combination.
                                                                                                                                                                                                                                               // Include all last letters and compare the
                                                                                         MD5((byte *) str, strlen(str), hash2);
                                                         if(strncmp((char*)hash1, (char*)hash2,
                                                                                                                                                                                                                                                                                                                                                                  iterate(hash1, hash2, str, idx + 1, len,
                                                                                                                                                                                                                                                                                                                                                                                                                             str[idx] = letters[c];
                                                                                                                    str[idx] = letters[c];
                                                                                                                                                                                                                                                                                                                                                                                                       // Recursive call
printf("found: %s\n", str);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 void strHex_to_byte(char * str, byte * hash){
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            * Convert hexadecimal string to hash byte.
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 char * pos = str;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      for (i = 0; i < MD5_DIGEST_LENGTH/sizeof *hash; i++) {</pre>
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       int i;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                pos += 2;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              sscanf(pos,
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              "%2hhx", &hash[i]);
```

Problem C

HopByte

The HopByte metric measures how much data has to travel through a network (hops × bytes). For an application executed in a parallel platform with a specific mapping of processes to processors, we can compute the HopByte metric as the sum (for all pairs of processes) of the product of the number of bytes exchanged by the distance (number of network hops) between the processors where the processes are mapped.

For our specific case, consider:

- The application as a directed weighted graph G = (V, E, w) with V the set of vertices (processes), E the set of edges (ordered pairs of vertices, or communication between processors), and $w: E \to \mathbb{N}^+$ the weight of the edges (number of bytes sent from one process to the other).
- The parallel platform as a set of processors P. The network topology of the platform is organized as a tree with height h, where the leaves represent P and the nodes in each level have the same number of children. The distance between processors (in number of hops) is computed as $d: P \times P \to \mathbb{N}$.
- The mapping of the application processes to processors as $\delta: V \to P$.

In this case, the HopByte (*HB*) metric is computed as:

$$HB = \sum_{i,j \in V} d(\delta(i), \delta(j)) w(i,j)$$

Input

The input has three sections to be read with no separation (whitespace, new line, etc).

The first section represents the application. Its first line contains two values: the first is the number of vertices (processors) |V| and the second is the number of edges (communication between processors) |E|. The other |E| lines contain three values: two vertices (whose ids are in the interval [0, |V|]) composing an edge and the weight of said edge.

The second section represents the network topology of the parallel platform (as a tree). Its

first line contains two values: the first is the number of leaves in the tree (processors) |P| and the second is the height of the tree h. The other h-1 lines contain a single value representing the number of children each node of that level has (number of children of the root node, number of children of the children of the root node, etc.).

The third section represents the mapping of processes to processors. The first line contains two values: the first is the number of processes |V| and the second is the number of processors |P|. The following V lines contain the id of a process in the interval [0, |V|] and the id of the processors where it is mapped in the interval [0, |P|].

The input must be read from the standard input.

Output

The output contains only one line printing the computed HopByte metric.

The output must be written to the standard output.

Input example 1	Output example 1
3 4 0 1 40 1 0 20 1 2 30 2 1 50 6 3 2 3 3 6 0 5 1 1	560
2 3	

```
topology, height);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 uint32_t processes, uint32_t** topology, uint32_t height){
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    uint32_t** topology, uint32_t height){
                                                                               where i and j are mapped
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        uint64_t hopbyte(uint32_t* mapping, uint32_t* application,
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           9
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      uint32_t find_hops(uint32_t first_node, uint32_t second_node,
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 /* Computes the hopbyte metric for the mapping st/
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           Computes the number of hops between two leaf nodes in
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          the direction of the root
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       common node)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                return common_level*2;
                                                                                                                                                                                                                                                                                                                                                                                                                                             uint64_t total = 0;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     while(first_node != second_node){
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             uint32_t common_level = 0;
                                                                                                                                                                                                                                                                                 for(i = 0; i < processes; ++i){</pre>
                                                                                                                                                                                                                                                                                                                                                               uint32_t i, j, bytes, hops;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       uint32_t current_level = height-1;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   //In a tree, we have two hops for each level that we
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                // Number of levels searched to find the common parent
                                                                                                                                                                                                                                                                                                                      //For all pairs of processors
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     // If the nodes are different, searches for their common
                                                                                                                                                                                                                                           for(j = 0; j < processes; ++j){
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       ++common_level;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              second_node = topology[current_level][second_node];
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     first_node = topology[current_level][first_node];
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   --current_level;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                // Move one level closer to the root
                                                                                                                                                             bytes = application[i*processes+j];
                                         hops = find_hops(mapping[i], mapping[j],
                                                                                                                                                                                                    //{\sf Gets} the number of bytes sent from i to j
                                                                                                                      /Gets the number of hops between the processors
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     go
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 topology, height));
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     int main (int argc, char* argv[]){
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                output
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                // Main program - reads input, computes hopbyte, prints
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            application = read_application(&processes);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              uint32_t processes, processors, height;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     uint32_t *application, **topology, *mapping;
                                                                                                                                                                                                                                                                                                                                                                                                        return 0;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     printf("%lu\n",hopbyte(mapping, application, processes,
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              mapping = read_mapping(processes, processors);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              topology = read_topology(&processors, &height);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         return total;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     //Tries to read the second input (topology)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        //Tries to read the third input (mapping)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   //Tries to read the first input (application)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 //Computes and outputs the hopbyte metric of the mapping
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    total+= bytes*hops;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             //Adds their product to
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           the hopbyte total
```

Problem D

Galaxy Simulator

From Wikipedia:

Newton's law of universal gravitation states that every particle attracts every other particle in the universe with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centers

The Newton's law leads up to a $O(n^2)$ time complexity algorithm, since the force of every particle must act upon all other particles, when one try to simulate a particle system. Such algorithm is too much slow to simulate a large number of particles, such as galaxies that are commonly composed of billions of stars. As an example, the Milky Way galaxy alone is supposed to have between 100 and 400 billion stars.

Barnes-Hut¹ came up with $O(n \log n)$ time complexity algorithm that enables one to approximate particle interaction by assuming that nearby bodies, a group, work as a single large fictitious body in the center of mass of the group. For a 2D galaxy simulation (where all stars share the same plane), the Barnes-Hut algorithm divides the space in four quadrants. Depending on the number of stars, each quadrant can be divided again in four quadrants, and so on. A tree data structure is used to keep all information: each node has four children, each one to represent a quadrant of the two dimensional space associated with the node. The particle interaction uses that tree data structure to compute approximations by considering the center of mass of each quadrant.

Input

An input represents only one test case. The first line contains one integer value N that represents the number of stars. Each one of the following N lines contains five floating-point values that describe the *mass*, the plane position (x, y) and the plane velocity (u, v) of each star. The last line contains the number of time steps that is used into this galaxy simulation.

¹Josh Barnes and Piet Hut. A hierarchical o (n log n) force-calculation algorithm. Nature, 324(6096):446, 1986.

The input must be read from the standard input.

Output

The output contains two lines: the first line contains the total vector velocity (u, v) considering all stars, in both dimensions; and the second line contains the center (x, y) of mass considering all stars.

The output must be written to the standard output.

```
Input

3
1.000000 0.334531 0.532345 -5.192624 -6.640964
1.000000 0.558670 0.453306 3.576856 1.123553
1.000000 0.573755 0.553912 -4.322105 1.478229
1000

Output for the input
-13.109914 -6.611076
0.509590 0.580083
```

```
//Undate velocities and positions
for(int i = 0; i < N; 1++) (int i = 0; i < N; 1++) (double ax = force_x[i]/mass[i];
double ay = force_y[i]/mass[i];
u[i] += ax*dt;
v[i] += u[i]*dt;
y[i] += v[i]*dt;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    void time_step(void) {
    //Allocate memory for root
    root = malloc(sizeof(struct node_t));
    set_node(root);
    root->min_x = 0; root->max_x = 1; root->min_y = 0; root->max_y = 1;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            void put_particle_in_tree(int new_particle, struct node_t *node) {
    //If no particle is assigned to the node
    if(inode-)has_particle) {
        node-particle = new_particle;
        node-particle = new_particle;
        node->has_particle = 1;

                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       calculate_mass(root);
calculate_center_of_mass_x(root);
calculate_center_of_mass_y(root);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         //Put particles in tree
for(int i = 0; i < N; i++) {
  put_particle_in_tree(i, root);</pre>
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             else if(!node->has_children) {
    //Allocate and initiate children
    //Allocate and initiate children
    node->children = malloc(*sizeof(struct node_t));
    for(int i = 0; i < 4; i++) {
        set_node(&node->children[i]);
    }
}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              free_node(root);
free(root);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     update_forces();
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             //Calculate mass and center of mass
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 //If the node has no children
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    //Calculate forces
//Put new particle into the appropriate child
place_particle(new_particle, mode);
//It now has children
node->has_children = 1;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          //Set boundaries for the children node->children[0].max_x = (node->min_x + node->max_x)/2; node->children[0].min_y = node->min_y; node->children[0].max_y = (node->min_y + node->max_y)/2; node->children[0].min_y = node->min_y; node->children[0].max_y = (node->min_y + node->max_y)/2; node->children[0].max_y = (node->min_y + node->min_y + node->max_y)/2; node->children[0].max_y = (node->min_y + node->min_y 
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       bounce(&x[i], &y[i], &u[i], &v[i]);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  /* This of course doesn't make any sense physically, 
* but makes sure that the particles stay within the 
* bounds. Normally the particles won't leave the 
* bounds. Normally the particles won't leave the 
* particles 
                                                                                                                                                                                                                                                                                                                                                                                   place_particle(node->particle, node);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              node->children[3].min_x = (node->min_x + node->max_x)/2; node->children[3].max_x = node->max_x; node->children[3].min_y = (node->min_y + node->max_y); node->children[3].min_y = node->max_y;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              node->children[1].min_x = (node->min_x + node->max_x)/2; node->children[1].max_x = node->max_x; node->children[1].min_y = node->min_y; node->children[1].max_y = (node->min_y + node->max_y)/2;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                   //Put old particle into the appropriate
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      area anyway.
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          void place_particle(int particle, struct node_t *node) {
    if(X[particle] <= (node->min_y + node->max_y)/2 && y[particle] <= (node->min_y + node->max_y)/2) {
        put_particle_in_tree(particle_s *node->nil_tren([i));
        else if(x[particle] > (node->min_x + node->max_y)/2 && y[particle] < (node->min_y + node->max_y)/2) {
        put_particle_in_tree(particle_s *node->rhildren[i]);
    } else if(x[particle_in_tree(particle_s *node->nax_y)/2 && y[particle] > (node->min_y + node->max_y)/2) {
        put_particle_in_tree(particle_s *node->rhildren[i]);
        put_particle_in_tree(particle_s *node->children[i]);
    }
}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  node->c_y = y[node->particle];
} else {
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           * Puts a particle in the right child of a node with children.
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               ouble calculate_center_of_mass_y(struct node_t *node) {
   if(!node->has_children) {
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    ouble calculate_center_of_mass_x(struct node_t *node) {
   if(!node->has_children) {
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    return node->c_x;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       return node->c_y;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                return node->total_mass;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 ible calculate_mass(struct node_t *node) {
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              put_particle_in_tree(particle,
else {
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  node->total_mass = 0;
for(int i = 0; i < 4; i++) {
    node->total_mass += calculate_mass(&node->children[i]);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   //Put new particle into the appropriate child
place_particle(new_particle, node);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       double m_tot = 0;
for(int i = 0; i < 4; i++) {
   if(node-vchidren[i].has_particle) {
      node-vc_y += node-vchildren[i].total_mass*calculate_center_of_mass_y(&node-vchildren[i]);
   m_tot += node-vchildren[i].total_mass;
}</pre>
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            node->c_x = x[node->particle];
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        node->total_mass = mass[node->particle];
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              put_particle_in_tree(particle, &node->children[3]);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            node->c_y /= m_tot;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 node->c_x /= m_tot;
```

Problem E

FDM in Heat Equation

The heat equation is solved by derivatives in space and time, as shown below:

$$\frac{\partial T}{\partial t} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right)$$

Some numerical methods solve it based on approximate approach. One of these methods is called Finite Different Method – FDM – which resolves derivatives through finites differences. Thus, the numerical approximate based on FDM is given by:

$$u_{i,j,k}^{t+1} = \frac{\Delta t}{\Delta h^2} \alpha \begin{bmatrix} u_{i+1,j,k}^t + u_{i-1,j,k}^t + u_{i,j+1,k}^t + u_{i,j-1,k}^t \\ + u_{i,j,k+1}^t + u_{i,j,k-1}^t - 6 \times u_{i,j,k}^t \end{bmatrix} - u_{i,j,k}^{t+1}$$

where time and space are discretized in regular steps. To simplify the problem, we adopt the same space discretization in all axes.

The initial condition is given by a cube with zero Celsius degree. The cube measures x, y and z centimeters, relying on the input problem parameter. Each face of the cube is heated to a temperature of 100 Celsius degree. Thus, sequential code calculates how long all the cube will take to be in 100-Celsius degree, using the previous FDM equation with 1.0 for the material heat velocity propagation.

Input

An input consists of only one test case. The first line contains tow positive values for Δt and Δh (both ≤ 1.0). The second line contains three positive values (z, y, x) of a cube $(1 \leq z, y, x \leq 100)$.

The input must be read from the standard input.

Output

The output has only one line. It contains the number of time steps to make the entire cube at 100-Celsius degree.

The output must be written to the standard output.

Input example 1	Output example 1
0.01 0.25 10 10 20	14383

```
void mdf_heat(double ***
   left + right -
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     register unsigned int steps = 0;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             register int continued = 1;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       register double alpha = deltaT / (deltaH * deltaH);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            while (continued){
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    steps++;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                const double boundaries){
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  const double deltaT,
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         for (unsigned int i = 0; i < npZ; i++){</pre>
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              const double inErr,
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 const double deltaH,
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         const unsigned int npZ,
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 const unsigned int npY,
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        const unsigned int npX,
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      double ***
u1[i][j][k] = alpha * ( top + bottom + up + down + (6.0f * u0[i][j][k] )) + u0[i][j][k];
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             for (unsigned int j = 0; j < npY; j++){</pre>
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     for (unsigned int k = 0; k < npX; k++){</pre>
                                                                                                                                                                                                                                                                                                                                           if ((j > 0) && (j < (npY - 1))){
up = u0[i][j-1][k];
                                                                                                                                                                                                                                                                                                                                                                                                                        else left = u0[i][j][k-1];
                                                                                                                                                                                 if ((i > 0) && (i < (npZ - 1))){
top = u0[i-1][j][k];
                                                                                                                                                                                                                                                              else up = u0[i][j-1][k];
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            if ((k > 0) && (k < (npX - 1))){
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              register double bottom = boundaries;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         register double top
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   register double down
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                register double right = boundaries;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              register double left
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            register double up
                                                                                                        else top = u0[i-1][j][k];
                                                                                                                                                                                                                                                                                       }else if (j == 0) down = u0[i][j+1][k];
                                                                                                                                                                                                                                                                                                                                                                                                                                             bottom = u0[i+1][j][k];
}else if (i == 0) bottom = u0[i+1][j][k];
                                                                                                                                                                                                                                                                                                               down = u0[i][j+1][k];
                                                                                                                                                                                                                                                                                                                                                                                                                                                                       right = u0[i][j][k+1];
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     left = u0[i][j][k-1];
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                __restrict__ u1,
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        __restrict__ u0,
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                = boundaries;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           = boundaries;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              = boundaries;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     = boundaries;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   fprintf(stdout, "%u\n", steps);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          u1 = ptr;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        u0 = u1;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 double maxErr = 0.0f;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               double ***ptr = u0;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     for (unsigned int i = 0; i < npZ; i++){</pre>
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           double err = 0.0f;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  for (unsigned int j = 0; j < npY; j++){
for (unsigned int k = 0; k < npX; k++){
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         err = fabs(u0[i][j][k] - boundaries);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               if (err > inErr)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      continued = 0;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         maxErr = err;
```

Problem F

Closest Pair of Points

Given an array of n points in the 2D plane, the problem is to find out the closest pair of points in the array. This problem arises in a number of applications. For instance, it can be used to monitor airplanes that come too close together in an air-traffic control system, thus avoiding possible collisions. The following formula is used to determine the distance between two points p and q in a 2D plane:

$$||pq|| = \sqrt{(p_x - q_x)^2 + (p_y - q_y)^2}$$

The given sequential program calculates the distance between the closest pair of points in $O(n(\log n)^2)$ time using a Divide and Conquer strategy. Your task is to improve the performance of the program using parallel strategies.

Input

The input contains only one test case. The first line of input contains an integer N ($2 \le N \le 80,000,000$), which denotes the number of points. Each of the next N lines contains two floats X and Y (- $10,000,000 \le X$, $Y \le 10,000,000$ with exactly 3 digits after the decimal), which denote the coordinates of the i-th point. There is no coincident point in the input data. The input must be read from the standard input.

Output

The output must contain a single line with a float D, denoting the distance between the closest pair of points. D must contain exactly 6 digits after the decimal.

The output must be written to the standard output.

Input example 1	Output example 1
5 -5.000 2.000 -15.000 -7.000 0.000 0.000 6.000 3.000 2.000 4.000	4.123106

```
double solve(int 1, int r) {
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             inline double abs2(double a)
                                                                                                                                                                                                                                                                                                                                                                                   inline double distance(Point a, Point b) {
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      inline double sqr(double a) {
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    bool compY(const Point& a, const Point& b) {
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 bool compX(const Point& a, const Point& b) {
                                                                                                                                                                                                                                                                                                                                                                                                                                                                             return a * a;
                                                                                                                                                                                                         double dist;
                                                                                                                                                                                                                                    double minDist = INF;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              return a.y<b.y;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        if (abs2(a.y-b.y)<EPS)</pre>
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    if (abs2(a.x-b.x)<EPS)</pre>
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         return a;
                                                                                                                                             if(r-1+1 <= BRUTEFORCESSIZE){</pre>
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           return a.x<b.x;</pre>
                                                                                                                                                                                                                                                                                                                                                     return sqr(a.x - b.x) + sqr(a.y - b.y);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                (a < 0.0)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         return a.x<b.x;</pre>
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      return a.y<b.y;</pre>
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      return -a;
                                                                                                                 for(i=1; i<=r; i++){</pre>
                                                                                  for(j = i+1; j<=r; j++) {
                            if(dist<minDist){</pre>
                                                       dist = distance(point[i], point[j]);
minDist = dist;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   <u>i++)</u>{
                                                                                                                                                                                                                                                                minDist; j++){
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              for(i=m+1; i<=r && abs(point[i].x-point[m].x)<minDist;</pre>
                                                                                                                                                                                                                                                                                  for(i=1; i<k; i++){
   for(j=i+1; j<k && border[j].y - border[i].y <</pre>
                                                                                                                                                                                                                                                                                                                                                                                   sort(&border[1], &border[1]+(k-1), compY);
                                                                                                                                                                                                                                                                                                                                                                                                                                               if (k-1 <= 1) return minDist;</pre>
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 for(i=m-1; i>=1 && abs(point[i].x-point[m].x)<minDist; i-</pre>
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            int k = 1;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       minDist = (dL < dR ? dL : dR);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      double dR = solve(m,r);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 double dL = solve(1,m);
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              int m = (1+r)/2;
                            return minDist;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   border[k++] = point[i];
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      border[k++] = point[i];
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      return minDist;
                                                                                                                                                                                                     if (dist < minDist){</pre>
                                                                                                                                                                                                                                  dist = distance(border[i], border[j]);
                                                                                                                                                                            minDist = dist;
```